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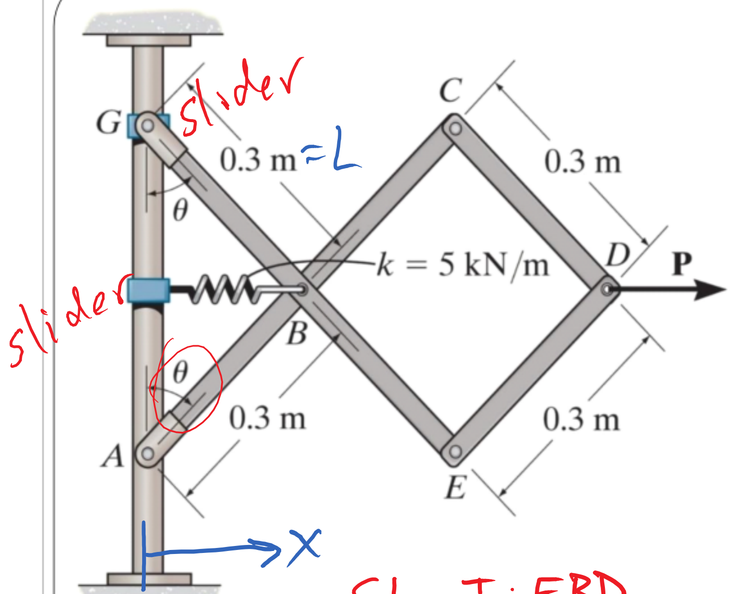
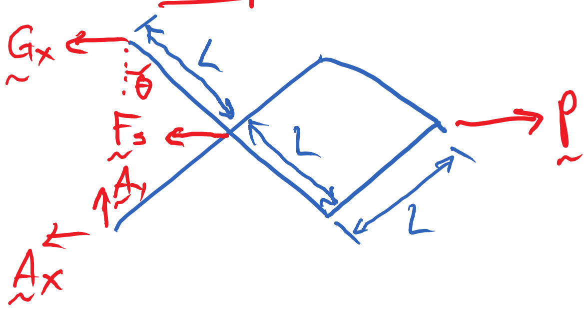
Tuesday, April 18, 2017 11:55 PM

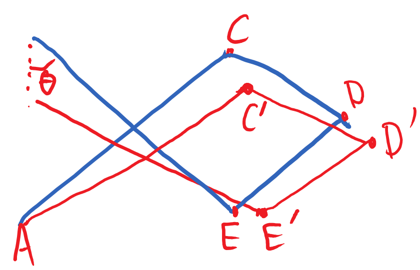


Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.

Step I: FBD



Step II: Virtual Displacements



Which forces are active?

~~A)~~ ~~A_x~~ — at a pin ⇒ no displacement ⇒ No virt. work

~~B)~~ ~~G_y~~ — is no G_y

C) F_s — spring can extend horizontally

~~D)~~ G_x — no horizontal displacement of G ⇒ no work by G_x

P is also active

Step III

Determine the location.

Where the active forces act.

Spring force: F_s acts @ point B

$x_B = L \cdot \sin\theta \quad (L = 0.3\,m)$

$\Rightarrow \delta x_B = L \cdot \cos\theta \cdot \delta\theta$

$$\delta x_B = \frac{\partial x_B}{\partial \theta} \cdot \delta\theta$$

Force P acts at D:

$x_D = 3L\sin\theta$

$\Rightarrow \delta x_D = 3 \cdot L \cdot \cos\theta \cdot \delta\theta$

Step IV

Apply principle of virtual work

$\delta U = P \cdot \delta x_D - F_s \cdot \delta x_B = 0$

spring force acts 180° to the virtual displacement

$$P \cdot (3 \cdot L \cdot \cos\theta \cdot \delta\theta) - F_s \cdot (L \cdot \cos\theta \cdot \delta\theta) = 0$$

$$\underbrace{(3 \cdot P - F_s)}_{=0} \underbrace{(L \cdot \cos\theta)}_{\neq 0 \text{ b.c. } \theta = 60^\circ} \cdot \underbrace{\delta\theta}_{} = 0$$

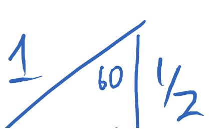
suppose $\delta\theta \neq 0$ -OR- just ignore

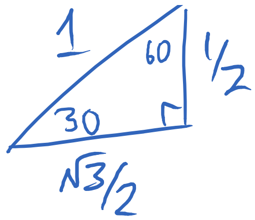
$$3P = F_s$$

$= k \cdot \Delta s$ — Δs = elongation of spring

$= k \cdot (s_{final} - s_{initial})$

$= k \cdot L \cdot [(\sin\theta_f) - (\sin\theta_i)]$





$$= k \cdot L \cdot \left[(\sin \theta_f) - (\sin \theta_i) \right]$$

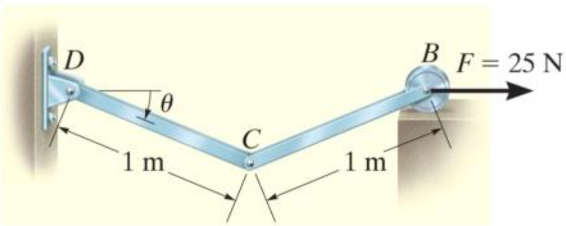
$$= k \cdot L \cdot \left[\sin(60^\circ) - \sin(30^\circ) \right]$$

$$P = \frac{kL}{3} \left[\frac{\sqrt{3} - 1}{2} \right]$$

$$= \frac{(5 \frac{kN}{m})(0.3\,m)}{3} \left[\frac{\sqrt{3} - 1}{2} \right]$$

$$= 0.183\,kN$$

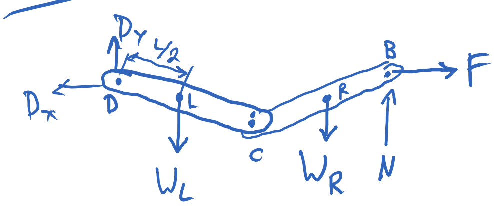
$$\boxed{P = 183\,N}$$



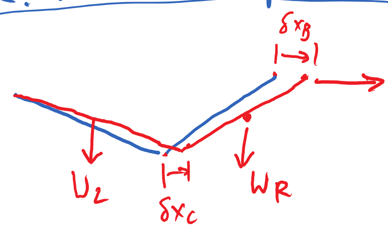
Determine the angle for equilibrium of the two-member linkage. Each member has a mass of 10 kg.

Solve for θ.

1. FBD



2. Virtual Displacements



3 active forces: W_L, W_R, F

3. Location of Active Forces

$$\underset{\sim}{X}_L = \frac{L}{2}\cos\theta \cdot \hat{i} - \frac{L}{2}\sin\theta \cdot \hat{j} \implies \delta \underset{\sim}{x}_L = -\frac{L}{2}\sin\theta\, \delta\theta\, \hat{i} - \frac{L}{2}\cos\theta \cdot \delta\theta\, \hat{j}$$

$$\underset{\sim}{X}_R = \frac{3L}{2}\cos\theta\, \hat{i} - \frac{L}{2}\sin\theta\, \hat{j} \implies \delta \underset{\sim}{x}_R = -\frac{3L}{2}\sin\theta\, \delta\theta\, \hat{i} - \frac{L}{2}\cos\theta \cdot \delta\theta\, \hat{j}$$

$$\underset{\sim}{X}_B = 2L\cos\theta\, \hat{i} \implies \delta \underset{\sim}{x}_B = -2 \cdot L \cdot \sin\theta \cdot \delta\theta \cdot \hat{i}$$

4. Virtual Work

$$\delta U = \underset{\sim}{W}_L \cdot \delta \underset{\sim}{x}_L + \underset{\sim}{W}_R \cdot \delta \underset{\sim}{x}_R + \underset{\sim}{F} \cdot \delta \underset{\sim}{x}_B = 0$$

$$(-mg\,\hat{j}) \cdot \left(-\frac{L}{2}\sin\theta\, \delta\theta\, \cancel{\hat{i}} - \frac{L}{2}\cos\theta\, \delta\theta\, \hat{j}\right)$$

$$+ (-mg\,\hat{j}) \cdot \left(-\frac{3L}{2}\sin\theta\, \delta\theta\, \cancel{\hat{i}} - \frac{L}{2}\cos\theta \cdot \delta\theta\, \hat{j}\right)$$

$$+ (F\hat{i}) \cdot (-2 \cdot L \cdot \sin\theta \cdot \delta\theta\, \hat{i}) = 0$$

$\hat{j} \cdot \hat{j} = 1$; $\hat{j} \cdot \hat{i} = 0$

$\hat{i} \cdot \hat{i} = 1$

$$\delta U = (-m \cdot g)\left(-\frac{L}{2}\cos\theta\, \delta\theta\right) + (-mg)\left(-\frac{L}{2}\cos\theta\, \delta\theta\right) + F \cdot (-2L\sin\theta\, \delta\theta) = 0$$

$$mg \cdot L\cos\theta \cdot \delta\theta - 2FL\sin\theta \cdot \delta\theta = 0$$

$$(m \cdot g \cdot \cos\theta - 2F\sin\theta) \cdot L \cdot \delta\theta = 0$$

$= 0 \qquad \neq 0$

$$\frac{m \cdot g \cdot \cos\theta - 2F \cdot \sin\theta}{\cos\theta} = 0$$

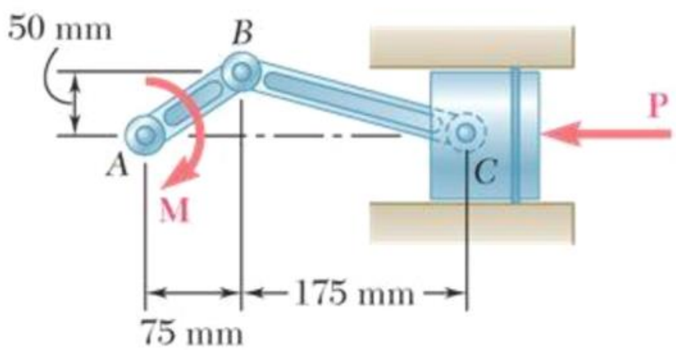
$$m \cdot g - 2F \cdot \tan\theta = 0$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{m \cdot g}{2 \cdot F}\right)$$

$$= \tan^{-1}\left(\frac{(10\,kg)(9.81\,m/s^2)}{2(25\,N)}\right)$$

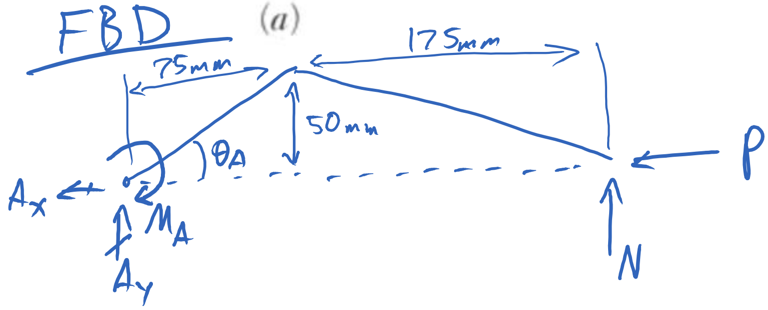
$$= 62.99^\circ$$

$$\boxed{\theta = 63^\circ}$$

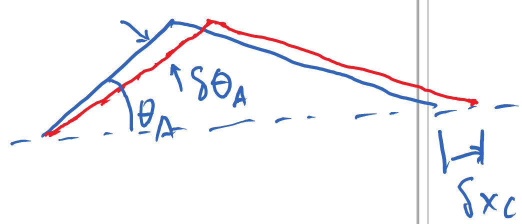


A couple **M** of magnitude 1.5 kN · m is applied to the crank of the engine system shown. ~~For each of the two positions shown,~~ determine the force **P** required to hold the system in equilibrium.

FBD



Virtual Displacements



$\underset{\sim}{M}_A$ is active

$\underset{\sim}{P}$ is active

$$\delta U = \underset{\sim}{M}_A \cdot \delta\underset{\sim}{\theta}_A + \underset{\sim}{P} \cdot \delta\underset{\sim}{x}_c = 0$$

$$\underset{\sim}{M}_A = -M_A \hat{k} \qquad \underset{\sim}{\theta}_A = \theta_A \hat{k} \Rightarrow \delta\underset{\sim}{\theta}_A = \delta\theta_A \hat{k}$$

$$\underset{\sim}{P} = -P\hat{i} \qquad \underset{\sim}{x}_c = x_c \hat{i} \Rightarrow \delta\underset{\sim}{x}_c = \delta x_c \hat{i}$$